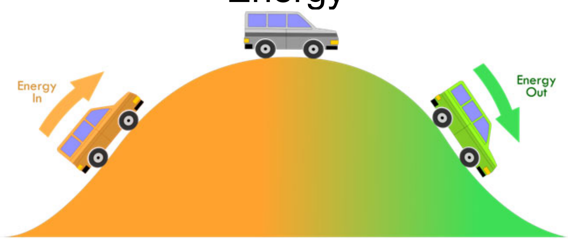


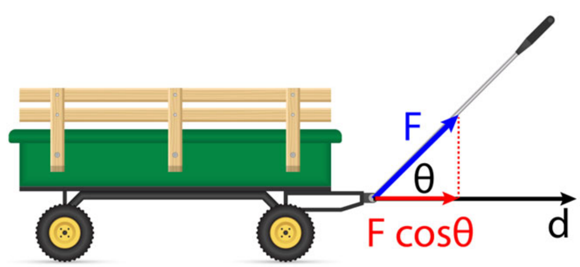
LuckySoul (Adobe Stock)

Work

- The work done on a system by a constant force is defined to be the product of the component of the force in the direction of motion and the distance through which the force acts.

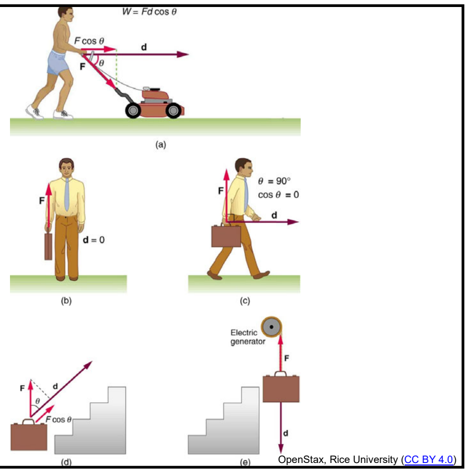
$$W = F_{\parallel} d$$

Units: Joules (J)



$$W = Fd\cos\theta$$

Ovidiu (Adobe Stock)



Example

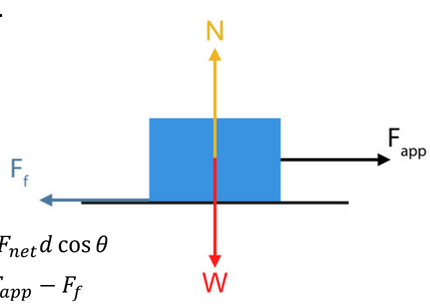
- A 50.0 kg crate is pulled 40.0 m along a horizontal floor by a constant force of 100.0 N at an angle of 37° from the horizontal. Calculate the work done on the crate.

$$W = Fd\cos\theta$$

$$W = (100)(40)\cos 37$$

$$W = 3190\ \text{J}$$

- If more than one force acts on an object, the net work is calculated using the net force.

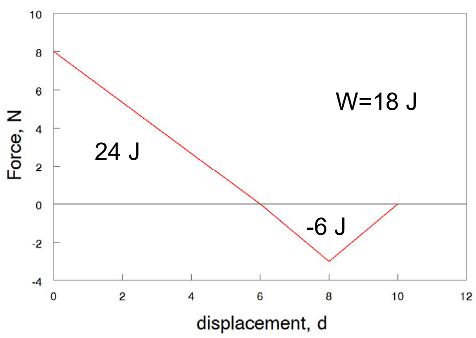


$$W_{net} = F_{net}d\cos\theta$$

$$F_{net} = F_{app} - F_f$$

$$W_{net} = (F_{app} - F_f)d\cos\theta$$

- If the force varies, then the work must be calculated from the area under a force vs. displacement curve.

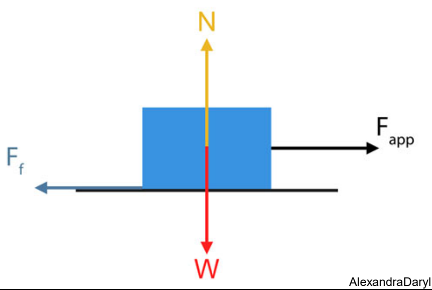


Energy

- When work is done on a system, energy is added to the system.
- The energy could be stored (potential energy) or cause motion (kinetic energy).
- Work done by friction leaves the system in the form of heat.

Kinetic Energy

- Consider a box being pushed along a horizontal surface with friction.



AlexandraDaryl (Adobe Stock)

- The net force arises from the applied force F_{app} and the friction force F_f.
- Since the net force is parallel to the displacement, the net work is

$$W_{net} = F_{net}d$$

- The result of the net force is an acceleration from v_0 to v.
  - The kinetic energy will increase.
- Substituting for F_{net} gives

$$W_{net} = mad$$

- We can use a kinematic equation to substitute for acceleration.

$$v^2 = v_0^2 + 2a(x - x_0) \qquad x - x_0 = d$$

$$a = \frac{v^2 - v_0^2}{2d}$$

$$W_{net} = m\left(\frac{v^2 - v_0^2}{2d}\right)d$$

- Rearranging this equation gives

$$W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- This expression is called the work-energy theorem, and it applies even for forces that vary in direction and magnitude.
- The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

- The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational kinetic energy.

$$E_k = \frac{1}{2}mv^2$$

- Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together.

Example

- A 145 g baseball is thrown with a speed of 25 m/s.
  a) What is the kinetic energy of the ball?
  b) How much work was done on the ball to make it reach this speed if it started from rest?

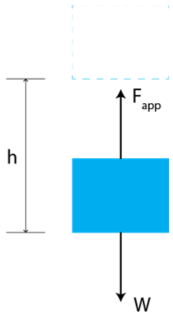
a) $K = \frac{1}{2}mv^2$

$$K = \frac{1}{2}(145 \times 10^{-3})(25)^2 = 45 \text{ J}$$

b) $W_{net} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

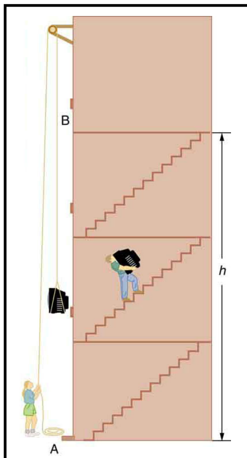
$$W_{net} = 45 \text{ J}$$

Gravitational Potential Energy



- An object of mass m is lifted through a height h at a constant speed.
- The force required to lift the object is equal to its weight mg.
- The work is therefore $W = mgh$
- This is defined to be the gravitational potential energy.

$$E_g = mgh$$



- The change in gravitational potential energy depends only on the change in height and not on the path.

$$\Delta E_g = mgh$$

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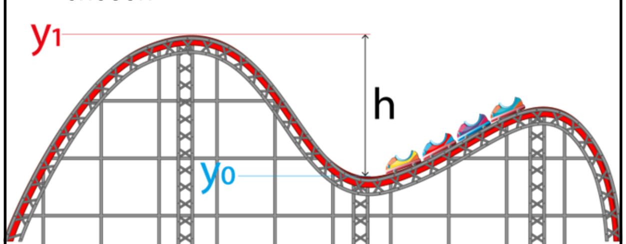
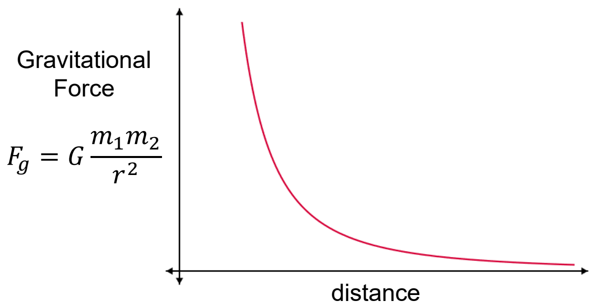
- The change in gravitational potential energy is also independent of the baseline chosen.



$$\Delta E_g = mg\Delta y$$

GraphicsRF (Adobe Stock)

- This expression applies for small distances where the value of g remain constant.
- We need an expression that works over distances such that g is not constant.
- This is necessary to correctly calculate the energy needed to place satellites in orbit or to send them on missions in space.
- We can calculate the work done (and thus potential energy) by finding the area under a force vs distance curve.



$$\text{Area} = W = U = Gm_1m_2\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

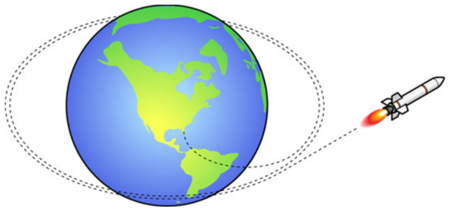
- We define gravitational potential energy as

$$E_g = -\frac{GMm}{r}$$

- Note:
  - Force at an infinite distance is 0.
  - Therefore, potential energy at an infinite distance must be 0.
  - Potential energy decreases as the distance decreases.

Escape Velocity

- The velocity necessary to escape the gravitational field of a planet (or any massive object).



Credit: Zern Liew (Adobe Stock Photo)

Calculating Escape Velocity

- The total energy of a moving object, *m*, near a large stationary mass, *M*, is

$$E_{total} = \frac{1}{2}mv^2 - G\frac{mM}{r}$$

- A long distance away (∞), the object will only have kinetic energy.
- On the surface of M, the object will only have gravitational potential energy.

- If the total energy is
  - >0, the mass escapes and never returns.
  - <0, the mass moves out a certain distance but is pulled back.
  - =0, the mass just escapes.
- The escape velocity is the velocity required to just escape the gravitational force of the Earth.
  - Total energy equals zero.

$$\frac{1}{2}mv^2 - G\frac{mM}{r} = 0$$

- The velocity is the escape velocity.

$$v = \sqrt{\frac{2GM}{r}}$$

- The escape velocity can also be given in terms of gravitational field strength.

$$g = G\frac{M}{r^2}$$

$$v = \sqrt{2gr}$$

Example

- Calculate the escape velocity from the surface of the earth.
  - $m_{earth} = 5.98 \times 10^{24}$ kg
  - $r_{earth} = 6.38 \times 10^{6}$ m

$$\frac{1}{2}mv^2 - G\frac{mM}{r} = 0$$

$$v = \sqrt{\frac{2GM}{r}}$$

$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(5.98 \times 10^{24})}{6.38 \times 10^{6}}}$$

$$v = 1.1 \times 10^{4}\ \text{m/s}$$

$$v = \sqrt{2gr}$$

$$v = \sqrt{2(9.8)(6.38 \times 10^{6})}$$

$$v = 1.1 \times 10^{4}\ \text{m/s}$$

Potential Energy

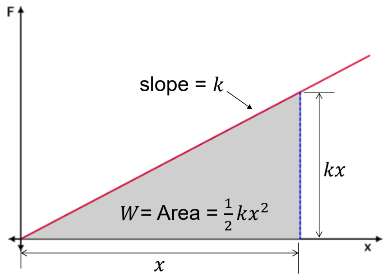
- Potential energy is the energy a system has due to position, shape, or configuration.
- It is stored energy that is completely recoverable.

Conservative Forces

- A conservative force is one for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.
- We can define a potential energy for any conservative force.
- The work done against a conservative force to reach a final configuration is the potential energy added.

Elastic Potential Energy

- The work done to stretch a spring a distance x is $W = Fx$.
- However, the force is not constant.
  - Hooke's law: $\vec{F}_s = -k\vec{x}$
- The work, therefore, is the area under the force vs displacement curve for the spring.



The potential energy is the work done on the spring.

$$E_s = \frac{1}{2}kx^2$$

Conservation of Mechanical Energy

- The work-energy theorem states that the net work done by all forces acting on a system equals its change in kinetic energy.
- If a conservative force, such as the gravitational force or a spring force, does work, the system loses potential energy.

- Therefore

$$-\Delta U = \Delta K$$

or

$$K_i + U_i = K_f + U_f$$

- This means that the total kinetic and potential energy is constant for any process involving only conservative forces.
- This is known as the conservation of mechanical energy principle.
  - The total kinetic plus potential energy of a system is defined to be its mechanical energy.

Example

- A 500 g ball is rolled along a level surface with a speed of 3.0 m/s. It goes up a 0.25 m ramp and compresses a spring (k=20.0) a distance x. Calculate x.

Ball – sabrog (openclipart)
Spring – algotruneman (openclipart)

$$K_i + U_i = K_f + U_f$$

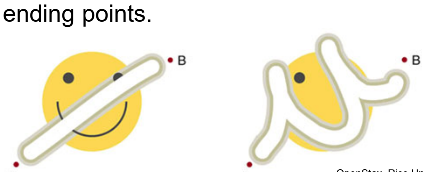
$$\frac{1}{2}mv^2 = mg\Delta y + \frac{1}{2}kx^2$$

$$x = \sqrt{\frac{\frac{1}{2}mv^2 - mg\Delta y}{\frac{1}{2}k}}$$

$$x = \sqrt{\frac{\frac{1}{2}(0.5)(3)^2 - (0.5)(9.8)(0.25)}{\frac{1}{2}(20)}} = 0.32 \text{ m}$$

Nonconservative Forces

- A nonconservative force is one for which work depends on the path taken.
- Friction is a nonconservative force.
  - Work done against friction depends on the length of the path between the starting and ending points.



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- Because of this dependence on path, there is no potential energy associated with nonconservative forces.
- Work done by a nonconservative force adds or removes mechanical energy from a system.
  - Friction creates thermal energy that dissipates, removing energy from the system.
  - Furthermore, even if the thermal energy is retained or captured, it cannot be fully converted back to work, so it is lost or not recoverable in that sense as well.

- When nonconservative forces are acting on the system the work-energy theorem states that the change in mechanical energy is equal to the work done by the nonconservative forces.

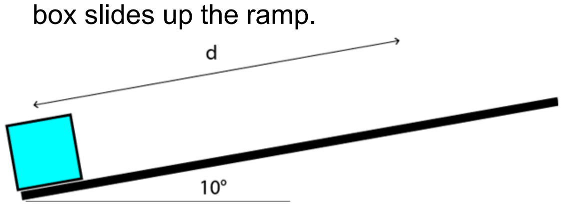
$$W_{nc} = \Delta K + \Delta U$$

or

$$K_i + U_i + W_f = K_f + U_f$$

Example

- A 1.0 kg box slides up a 10° ramp. The box has an initial velocity of 2.5 m/s and the force of friction between the box and ramp is 5.0 N. Calculate the distance the box slides up the ramp.



$$K_i + U_i + W_f = K_f + U_f$$

$$\frac{1}{2}mv^2 - F_f d = mgh$$

$$\frac{1}{2}mv^2 - F_f d = mgd\sin\theta$$

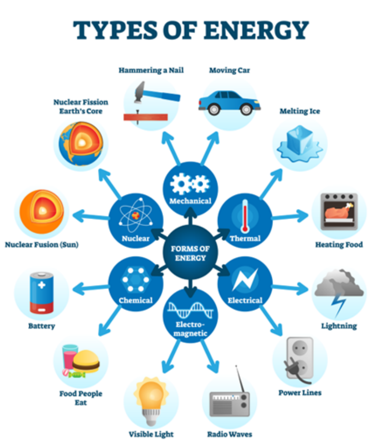
$$d = \frac{\frac{1}{2}mv^2}{mg\sin\theta + F_f}$$

$$d = \frac{\frac{1}{2}(1)(2.5)^2}{(1)9.8\sin 10 + (5)} = 0.47\text{ m}$$

Law of Conservation of Energy

- Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

$$\Sigma E_i = \Sigma E_f$$



VectorMine (Adobe Stock)

Power

- Power is the rate at which work is done.

$$P = \frac{W}{\Delta t}$$

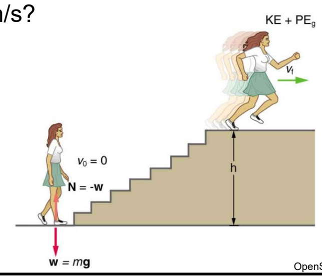
- Since work is energy transfer, power is also the rate at which energy is expended.

$$\boxed{P = \frac{\Delta E}{\Delta t}}$$

Units: Watts (W)

Example

What is the power output for a 60.0 kg woman who runs up a 3.00 m high flight of stairs in 3.50s, starting from rest but having a final speed of 2.00 m/s?



$$P = \frac{\Delta E}{\Delta t} \qquad \Delta E = (K_f + U_f) - (K_i + U_i)$$

$$P = \frac{K_f + U_f}{\Delta t} = \frac{\frac{1}{2}mv^2 + mg\Delta y}{\Delta t}$$

$$P = \frac{\frac{1}{2}(60)(2)^2 + (60)9.8(3)}{(3.5)} = 538\ \text{W}$$

Efficiency

- Even though energy is conserved in an energy conversion process, the output of useful energy or work will be less than the energy input.
- The efficiency of an energy conversion process is defined as

$$Efficiency = \frac{useful\ energy\ or\ work\ output}{total\ energy\ in} = \frac{W_{out}}{E_{in}}$$